



BEAMEM

A Research on Mutual Interaction of Two Electron Beams via Space Charge

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= a changed initial value

12-13-99 Rev 2
11-29-04 Rev 3 updated identifiers
12-22-2005 Logo changed

1. Introduction

The purpose of this study was to look into the strength of the assumed interaction of two electron beams' space charge in a cathode ray tube monitor. The phenomenon is observed as a repulsion of two beam's pixel images on the screen. As one beam is turned on/off the other beam moves a few tens of micrometers at least.

Crude model: A single electron travels alongside another electron beam after the gun section. We study the perpendicular forces affecting the electron only during the rest of the flight towards the screen. Most effects in the transverse direction are based on space charge. Most other effects can be neglected in this approximation. Deflecting magnetic fields are probably not strong enough to be significant.

We assume that the probe electron is first in rest (transverse motion only). The field produced by another nearby beam's space charge forces the electron to move away from it. It will be effective during the time-of-flight of the electron. When hitting the screen the distance difference gained is locked and can be readily observed. We can study a single electron as a good approximation for this situation. To handle two beams would be more elaborate and would probably not increase accuracy significantly.

We assume the following physical situation. The electron travels along a beam at a distance x_0 in a parallel direction (refer to Figure 1 below) a distance of L (meters). The dimensions of the beam are changing during the flight but the changes are so small compared to the distance between the beams, that they are ignored. The difference in distance generated x_d , is small compared to the distance x_0 . The electron has an initial velocity which it has gained in the gun. When it is released to the "free" space there practically only the anode voltage to affect it. The perturbation is from the nearby beam only. The force equation is a Newtonian causing an acceleration cross the beams. The acceleration is quadratic in relation to time as usual. Thus the highest sensitivity to this effect is at the beginning of the free space region where most of the time-of-flight is spent. We can assume that the distance between the beams there is a good estimate for the effective distance.

In practice, there may be more than one perturbing beam. Then the resulting deviation is calculated as a vectorial sum from the gun geometry. Another detail is the fact that the beams are not parallel, quite the opposite. They are focused into a single point at the screen. The deflecting forces are highest close to the screen but, on the other hand, the effective interaction time is much shorter than at the beginning.

In this revised study the initial values have been checked with the aid of a real gun and all other initial values have been checked as well. A careful inspection of the gun sample revealed that there is a field-free space after the gun exit. Also, inside the gun there are a few small field-free spaces. The beams can affect each other while traveling through these spaces. Their effects on the time-of-flight must be taken into account (tf).

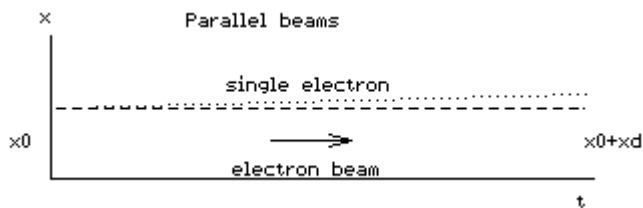


Figure 1. Position of electron beams

The deflecting effect of the beam is from the total space charge of the electron content of the beam. The charges form a line charge. Inside the beam the electric field goes to zero as a result of canceling effect of other electrons into each other. The repulsion does not affect the beam electrons themselves as strongly as one would think. The external field from the line charge behaves as $1/r$ instead of $1/(r^2)$ as would be expected in the first place. This means that the force is much stronger at longer distances than the inverse quadratic force. The range of the field is much longer also.

In the following, we will study the basic facts and numeric values of this model and we will prepare a mathematical model. Finally, we get some numerical results representing the deviation of the reference electron caused by the nearby beam.

2. Initial Values

Beam diameter (m)	beam current (A)	first accelerating voltage (V0)
$d := 0.0001$	$I_0 := 0.00035 \text{ #}$	$V_0 := 6000$
main accelerating voltage V1 - V0		$V_1 := 21000$

actual travel (m)

$$L := 0.34$$

initial distance of the electron from the beam (m)

$$x_0 := 0.00525$$

travel of the initial acceleration (m)

$$s := 0.05 \quad \#$$

natural constants

$$m_e := 9.109558 \cdot 10^{-31} \quad \# \quad (\text{kg})$$

$$e := 1.6022 \cdot 10^{-19} \quad \# \quad (\text{C})$$

$$\epsilon_0 := 8.85 \cdot 10^{-12} \quad \#$$

$$\pi := 3.1415926$$

field-free travel after the gun exit #

$$x_f := 0.030 \quad \#$$

3. Calculated Entities

duration of the first acceleration (s)

$$t_0 := s \cdot \sqrt{2 \cdot \frac{m_e}{e \cdot V_0}} \quad \#$$

$$t_0 = 2.177 \cdot 10^{-9}$$

end velocity after the first acc. (m/s)

$$v_0 := \sqrt{2 \cdot e \cdot \frac{V_0}{m_e}} \quad \#$$

$$v_0 = 4.594 \cdot 10^7$$

time-of-flight in the field-free space #

$$t_f := \frac{x_f}{v_0} \quad \#$$

$$t_f = 6.53 \cdot 10^{-10} \quad \#$$

time-of-flight with the first acceleration taken into account (s)

$$T := \frac{L \cdot \sqrt{2 \cdot m_e \cdot e \cdot V_1 + v_0^2 \cdot m_e^2} - v_0 \cdot L \cdot m_e}{e \cdot V_1} + tf *$$

$$T = 5.395 \cdot 10^{-9}$$

beam perveance after first accel.

$$P := \frac{I_0}{(V_0)^{1.5}} *$$

$$P = 7.531 \cdot 10^{-10}$$

number of electrons in the beam 1/sec

$$N := \frac{I_0}{e}$$

$$N = 2.184 \cdot 10^{15}$$

charge density of electrons, a line charge (C/m)

$$\rho := N \cdot \frac{T \cdot e}{L} *$$

$$\rho = 5.554 \cdot 10^{-12}$$

the electric field made by the line charge at x0

$$E_b := \frac{\rho}{2 \cdot \pi \cdot \epsilon_0 \cdot x_0} *$$

$$E_b = 19.024 \quad (\text{V/m})$$

the resulting accelerating force (N)

$$F := e \cdot E_b$$

$$F = 3.048 \cdot 10^{-18}$$

the equation of motion due to the transverse force, assuming the distance to stay approximately constant

$$\frac{d^2}{dt^2} x(t) = \frac{F}{me}$$

having a solution

$$x(t) := \frac{F \cdot t^2}{2 \cdot me} *$$

The deviation after the time-of-flight T

$$x_d := F \cdot \frac{T^2}{2 \cdot me} *$$

$$x_d = 4.87 \cdot 10^{-5}$$

or in practical dimensions (μm)

$$x := 1000000 \cdot x_d$$

$$x = 48.696 \quad @ I_0 = 350 \mu\text{A}$$

This is a rather reliable estimate for the beam deviation with with the initial dimensions, acquired gun dimensions and beam parameters.

4. A Practical Formula

$$x_d := e \cdot \frac{I_0}{2 \cdot \pi \cdot \epsilon_0 \cdot x_0} \cdot \frac{e \cdot V_1}{L} \cdot \left[\frac{L \cdot \sqrt{2 \cdot m_e \cdot e \cdot V_1 + 2 \cdot e \cdot V_0 \cdot m_e} - \sqrt{2 \cdot e \cdot \frac{V_0}{m_e} \cdot L \cdot m_e}}{e \cdot V_1} + t_f \right] \cdot e \cdot \left[\frac{L \cdot \sqrt{2 \cdot m_e \cdot e \cdot V_1 + 2 \cdot e \cdot V_0 \cdot m_e} - \sqrt{2 \cdot e \cdot \frac{V_0}{m_e} \cdot L \cdot m_e}}{e \cdot V_1} + t_f \right]$$

#

The deviation is directly proportional to the beam current

5. Conclusions

The formula gives a good estimate for the beam deviation. The greatest sensitivity to the effect is from the beginning of the travel in free space where the beams can see each other without any obstructions. That is the region of the anode voltage giving the final velocity to the electrons. It appeared that there are short regions inside the gun where the beams can "see" each other.

The sensitivity to the effect will diminish if the gun exit velocity of the electrons becomes higher. It means that the gun and focusing system end voltages should be rather high. Increasing the beam distance after the guns will diminish the effect. Quite effective is also the diminishing of the beam perveance (smaller current, higher voltage).

In appendix 4 we have presented a more accurate model for deviation requiring much more numerical work before the deviation itself can be calculated. The deviation figure will probably not increase much.

Note that the actual deviation is twice the calculated as the this phenomenon will affect both beams, of course. However, one can not observe beams with zero current and thus the practical deviation is as calculated.

The effect of the short free-field region is relatively small in this case as the beam has already been accelerated with a rather high voltage.

Appendix 1. References

- 1. H. F. Ivey, *Advances in Electronics and Electron Physics*, vol 6 137-256 (1954)**
- 2. J. R. Pierce, "Theory and Design of Electron Beams", D. Van Nostrand Co., New York, (1949)**
- 3. A. Septier, *Focusing of Charged Particles*, vol II, Academic Press New York, (1967)**

Appendix 2. Behavior of an Electron Beam in Vacuum, a Short Mathematical Treatment

An electron beam moving in vacuum can be described with the following equations assuming there are no other transverse forces affecting

$$m_e \cdot \frac{d^2}{dt^2} r = e \cdot \frac{I}{2 \cdot \pi \cdot r \cdot \left[\left(\frac{d}{dt} z \right) \cdot \epsilon_0 \right]}$$

$$m_e \cdot \frac{d^2}{dt^2} z = e \cdot E$$

We ignore all relativistic effects and the own magnetic field of the beam. Also the initial thermal velocities are ignored. E is the z-directional external electric field and r,z are the polar coordinates. I is the beam current. In order to calculate only the mutual repulsion of the electrons, one integrates:

$$\frac{d^2}{dt^2} r = \frac{e \cdot I}{2 \cdot \pi \cdot m_e \cdot r \cdot \left(\frac{e}{m_e} \cdot E \cdot t + \frac{d}{dt} z_0 \right)}$$

Here the derivative of z₀ means the z-directional initial velocity. If there is no external force affecting the beam the equation becomes

$$\frac{d^2}{dt^2} r = \frac{e \cdot I}{2 \cdot \pi \cdot m_e \cdot r \cdot \frac{d}{dt} z_0}$$

This can be solved approximately

$$\int_1^R \frac{1}{\sqrt{\ln(R)}} dR = \frac{z}{r_m} \cdot \sqrt{30000 \cdot K}$$

the perveance is
and

$$K = \frac{I}{V^{1.5}}$$

$$R = \frac{r}{r_m}$$

$2 \cdot r_m$ is the beam minimum waist diameter into which it will presumably converge as a focused beam.

As a result, we have a universal curve which is valid for all electron beams with the assumptions noted earlier. One only needs a scaling (refer to Septier /3/). The curve is symmetric around the minimum which means that one can not focus the beam indefinitely. If one tries, the waist may become smaller but the beam will diverge after the minimum much more strongly. Thus the length of the focused area becomes very short. One can not beat the Liouville theorem.

If one takes into account the thermal spreading of the beam, the beam behavior changes at long distances from the guns. It becomes from a beam with clear boundaries into a spread and diffuse having lots of fog around it. This does not affect our calculations here. The thermal spreading can be handled mathematically. The reason is the initial velocity of the electrons gained at the hot gun. The velocities are random both in amplitude and direction. The resolution development of the CRT will finally hit into this phenomenon.

Appendix 3. Other Factors Affecting the Deviation of a Pair of Beams.

Other factors may affect the beam deviation in addition to the space charge. Accurate estimates require measurements in the monitor electronics and knowledge of tube parameters. They seem to be outside of the range of this study. In the following we only mention other factors which may be strong enough.

1. Dependence of the anode high voltage from the beam currents (loading effect).
2. The loading effect of the video output driver stages to their own power sources with some feedback to other stages, like the deflection units.
3. Resistive effects on the screen surface caused by the beam return current and its effect on the electric field.
4. Some external synchronous magnetic field causing deflection to the beams. it could be from some deflection unit or power source or their wiring.

Some effect may be impossible due to wiring. The third effect may be very small.

As a repair to these effects is a good filtering and regulation of all power sources and separation of them in critical stages. A good EMC-practice is an excellent prevention.

Appendix 4. A More Accurate Calculation

This, a more accurate model of the deviation caused by the space charge field. We assume that this model does not bring too big changes to the deviation figure as estimated earlier. The change in the deviation is, however, always positive thus increasing it. It is due to the fact that the force becomes stronger when the beams become closer to each other. If the electron travels inside the other beam, the field will start to diminish quickly and becomes zero at the beam center. That requires a still better model which is not required at this time. Refer to Figure 2. for geometric details.

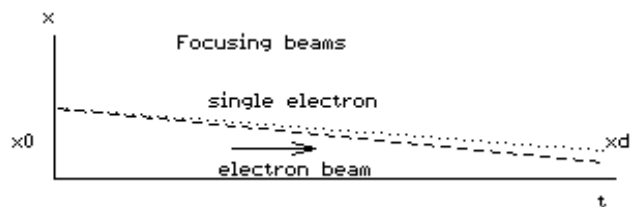


Figure 2. Positions of the beam and the reference electron

The equation of motion for the transverse force, if one assumes that the distance changes

$$\frac{d^2}{dt^2} x(t) = \frac{F(x(t))}{me} = \frac{\rho}{me \cdot 2 \cdot \pi \cdot \epsilon_0 \cdot x(t)}$$

where

$$Eb(x) = \frac{\rho}{2 \cdot \pi \cdot \epsilon_0 \cdot x(t)}$$

the corresponding accelerating force (N)

$$F(t) = e \cdot Eb(x)$$

and the equation becomes

$$C1 := \frac{\rho}{2 \cdot \pi \cdot \epsilon_0 \cdot me} *$$

$$x(t) \cdot \frac{d^2}{dt^2} x(t) = \frac{C1}{2}$$

$$C1 = 1.096 \cdot 10^{29}$$

we transform it with

$$\frac{d}{dt} x(t) = u(x)$$

$$\frac{d}{dx} u(x)^2 = \frac{C1}{x}$$

the integral becomes

$$u(x)^2 = C1 \cdot \ln(x) + C2$$

or

$$\frac{d}{dt} x(t) = \sqrt{C1 \cdot \ln(x(t)) + C2}$$

C2 is the integration constant

It is no coincidence that the mathematics of this model looks familiar (refer to appendix 2). There is no closed solution for this equation either. We need other methods.

substitute

$$x(t) = e^{\frac{w^2}{C1}} \cdot e^{-\frac{C2}{C1}}$$

to transform the equation

$$\int_{x0}^{x(t)} \frac{1}{\sqrt{C1 \cdot \ln(x) + C2}} dx = t + C3$$

C3 is an integration constant

$$2 \cdot \frac{e^{-\frac{C2}{C1}}}{C1} \int_{\sqrt{C1 \cdot \ln(x0) + C2}}^{\sqrt{C1 \cdot \ln(x(t)) + C2}} e^{\frac{w^2}{C1}} dw = t + C3$$

which is

$$\int_{\sqrt{C1 \cdot \ln(x0) + C2}}^{\sqrt{C1 \cdot \ln(x(t)) + C2}} e^{\frac{w^2}{C1}} dw = \frac{(t + C3) \cdot e^{\frac{C2}{C1}} \cdot C1}{2} \quad \text{if} \quad w = \sqrt{C1 \cdot z}$$

$$\int_{\sqrt{\ln(x0) + \frac{C2}{C1}}}^{\sqrt{\ln(x(t)) + \frac{C2}{C1}}} e^{(z^2)} dz = \frac{(t + C3) \cdot e^{\frac{C2}{C1}} \cdot C1}{2} + C3$$

This is one form of exponent integrals and also related to the erf-function (refer to below). It has no closed solution but a more elaborate numeric integration is required.

$$\operatorname{erf}\left(i \cdot \frac{\sqrt{\ln(x(t)) \cdot C1 + C2}}{\sqrt{C1}}\right) = i \cdot \frac{\left[(t + C3) \cdot \exp\left(\frac{C2}{C1}\right) \cdot C1 - i \cdot \sqrt{\pi} \cdot \operatorname{erf}\left(i \cdot \frac{\sqrt{\ln(x0) \cdot C1 + C2}}{\sqrt{C1}}\right) \right]}{\sqrt{\pi}}$$